

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1271

F

Unique Paper Code : 2352571201

Name of the Paper : ELEMENTARY LINEAR
ALGEBRA

Name of the Course : **B.Sc. (Prog.) DSC-B2**

Semester : II

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. **All questions** carry equal marks.

P.T.O.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that:

$$\|x + y\| \leq \|x\| + \|y\|$$

- (b) Define norm of a vector. Find a unit vector in the

same direction as the vector $\left[\frac{1}{5}, -\frac{2}{5}, -\frac{1}{5}, \frac{1}{5}, \frac{2}{5}\right]$.

Is the normalized (resulting) vector longer or shorter than the original? Why?

- (c) Use Gaussian elimination method to solve following systems of linear equations. Give the complete solution set, and if the solution set is infinite, specify two particular solutions.

$$\begin{aligned} 3x + 6y - 9z &= 15 \\ 2x + 4y - 6z &= 10 \\ -2x - 3y + 4z &= -6 \end{aligned}$$

2. (a) Determine whether the two matrices are row equivalent?

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 1 \\ 5 & -1 & 5 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 10 \\ 2 & 0 & 4 \end{bmatrix}$$

- (b) Find the rank of the following matrix.

$$\begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 4 & 0 & -2 & 1 \\ 3 & -1 & 0 & 4 \end{bmatrix}$$

- (c) Express the vector $X = [2, -5, 3]$ as a linear combination of the vectors $a_1 = [1, -3, 2]$, $a_2 = [2, -4, -1]$, and $a_3 = [1, -5, 7]$ if possible.

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3. (a) Determine the characteristic polynomial of the following matrix.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

- (b) Show that the set of vectors of the form $[a, b, 0, c, a - 2b + c]$ in $\mathbb{R}^5$ forms a subspace of $\mathbb{R}^5$ under the usual operations.

- (c) For $S = \{x^3 + 2x^2, 1 - 4x^2, 12 - 5x^3, x^3 - x^2\}$, use the Simplified Span Method to find a simplified general form for all the vectors in $\text{span}(S)$, where S is the given subset of P_3 , the set of all polynomials of degree less than or equal to 3 with real coefficients.

4. (a) Use the Independence Test Method to determine whether the given set S is linearly independent or linearly dependent.

$$s = \{1, -1, 0, 2\}, [0, -2, 1, 0], [2, 0, -1, 1\}$$

- (b) Let the subspace W of $\mathbb{R}^5$ be the solution set to the matrix equation $AX = 0$ where A is

$$\begin{bmatrix} 1 & 2 & 1 & 0 & -1 \\ 2 & -1 & 0 & 1 & 3 \\ 1 & -3 & -1 & 1 & 4 \\ 2 & 9 & 4 & -1 & -7 \end{bmatrix}$$

Find the basis and the dimension for W . Show that $\dim(W) + \text{Rank}(A) = 5$.

- (c) Show that P_n , the set of all polynomials of degree less than or equal to n with real coefficients, is a vector space under the usual operations of addition and scalar multiplication.

5. (a) Consider the mapping $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by

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$$f([a_1, a_2, a_3]) = [a_1, a_2, -a_3]$$

Prove that f is a linear transformation.

(b) Find the matrix for the linear transformation

$L: P_3 \rightarrow R^3$ given by

$$L(a_3x^3 + a_2x^2 + a_1x + a_0) = [a_0 + a_1, 2a_2, a_3 - a_0]$$

With respect to the bases $B = (x^3, x^2, x, 1)$ for P_3
and $C = (e_1, e_2, e_3)$ for R^3

(c) Consider the linear operator $L: R^n \rightarrow R^n$ given by

$$L([a_1, a_2, \dots, a_n]) = [a_1, a_2, 0, \dots, 0]$$

Find the kernel of L and range of L .

6. (a) Consider the linear transformation $L: R^3 \rightarrow R^3$ given

$$\text{by } L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 & 5 \\ -2 & 3 & -13 \\ 3 & -3 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find the basis for kernel of L .

- (b) Consider the linear operator $L: R^2 \rightarrow R^2$ given by

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Show that L is one-to-one and onto operator.

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(c) Consider the linear transformation $L: P_3 \rightarrow P_2$ given

by $L(p) = p'$ where $p \in P_3$

Is L an isomorphism?

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